## 2. Survey of Some Interpretations

One can solve the measurement problem by one of four (overlapping!) strategies. Each comprises various views: I shall say nothing about some of the views I list -- and there are many unlisted! First I state the measurement problem with more technicality.

### 1. The Problem

(1) Assume orthodox ascriptions of values to physical quantities: called the eigenstate-eigenvalue link.

(2) Schroedinger's Cat: the threat that indefiniteness in the micro-realm can be transmitted to the macro-realm, contradicting its apparent definiteness.

$$\varphi_i \otimes \psi_0 \rightarrow \varphi_i \otimes \psi_i$$

$$\text{So: } (\sum_i \lambda_i \varphi_i) \otimes \psi_0 \rightarrow \sum_i (\lambda_i \varphi_i \otimes \psi_i) =: \Psi$$

$\Psi$ resists being interpreted as yielding definite pointer-positions. For ...

(3) Can we secure this by partial tracing? By partial tracing $\Psi$ determines as the reduced state on the microsystem, the mixture: $\sum_i |\lambda_i|^2 |\varphi_i\rangle\langle\varphi_i|$. And $\Psi$ determines as the reduced state on the apparatus, the mixture: $\sum_i |\lambda_i|^2 |\psi_i\rangle\langle\psi_i|$. If we could give these mixtures the ignorance interpretation, then all would be well. But apparently we cannot: for if we could, then the composite system (or ensemble of such ) would also be in a mixture, not a pure state. So these reduced states are hard to interpret: 'improper mixtures'.

(4) The problem in (2,3) is not an artefact of the over-simple model of measurement, with its non-disturbance of the $\varphi_i$, its registration of $\varphi_i$ by eigenstate correlation, and its non-degenerate pointer-position. The 1960s saw a series of theorems enforcing the problem for more general models of measurement (Wigner 1963, Fine 1969, 1970; Brown 1986).

### 2. Survey of Solutions

One can solve the problem by one of four strategies:

(Physics) *Deny the thrust of the Cat argument; (and so secure a definite macrorealm while keeping the orthodox unitary dynamics of a strictly isolated system).*

Here I will concentrate on appeal to the environment; i.e. on using the fact that the system is not isolated. This will lead in to

(Physical Values) *Accept the thrust of the Cat argument; and keep the orthodox (unitary) dynamics of a strictly isolated system. Secure a definite macrorealm by positing new values, beyond those of the orthodox eigenvalue-eigenstate link;.*

Very different views fall under this strategy: in order of increasing radicalism, they are:
(i) the modal interpretations of Kochen-Healey-Dieks, van Fraassen, and Bub;
(ii) the ontological interpretations of de Broglie & Bohm; i.e. particle and wave
(iii) the Copenhagen interpretation: i.e. philosophical necessity of a classical realm;
(iv) the relative-state interpretation of Everett et al.: values for each measurement-outcome.

For all views, the extra values must satisfy three constraints:
(a) they need to be consistent with no-hidden-variable proofs: von Neumann-Gleason-Bell-KS
(b) they need to mesh with the empirical success of the naive Born-rule probabilities;
(c) they need to evolve over time st the definiteness of the macrorealm is maintained.

(Mental Values) *Accept the thrust of the Cat argument; and keep the orthodox (unitary) dynamics of a strictly isolated system. Allow an indefinite physical macrorealm, but secure the belief that it is definite by positing new 'mental values'.*

Here I will concentrate on two views.
(i) Consciousness collapses the wave-packet; (so unitary dynamics is retained only for the solely physical); (von Neumann, Wigner briefly; recently an analogue by Stapp).
(ii) Each brain supports many minds, each mind with one of the beliefs corresponding to a measurment-outcome. (Albert&Loewer; Lockwood).

(Dynamics) *Accept the thrust of the Cat argument; secure a definite macrorealm by denying the orthodox unitary dynamics of a strictly isolated system, and modelling the 'collapse of the wave packet' -- the evolution of a pure state (superposition) into a mixture.* (GRW, Pearle).
I turn to details of these strategies, in the order above.

(Physics) *Deny the thrust of the Cat argument; (and so secure a definite macrorealm while keeping the orthodox unitary dynamics of a strictly isolated system).*

Various possible grounds for the denial can be given. Two major approaches of this kind (again, overlapping!) are:

(Super) Invocation of superselection rules;

Two main general problems: dynamics and the stochastic transition.

(Decoherence) Invocation of the environment . Use the fact that the system and apparatus is not isolated to argue that a definite pointer position is obtained. There are two aspects to this: only the first can succeed without outside help.

(a) Models: In many plausible models of the interaction between an apparatus --indeed a general macrosystem -- and its environment, the weirdness of superposition is carried away very fast by e.g. air molecules or photons. There are two points here:

(i) 'weirdness .. away': That is: the total state (system+apparatus+environment) determines a state for the apparatus ('reduced state'), which is very close to the desired state -- very close to a mixture of pointer-position eigenstates. For short: the interference terms, measuring the difference between the actual state of the apparatus and the desired state, become small.

(ii) 'very fast'. Indeed, amazingly fast, even for a tiny apparatus/macrosystem. Consider a dust particle of radius $10^{-3}$ cm. in air: consider the interference terms distinguishing a superposition, of two positions for the (centre of the) dust particle, just $10^{-4}$ cm. apart, from the corresponding mixture. These terms converge to 0 like $\exp^{-10(\text{tothe}36)t}$ ! And they stay low for very long time-scales ($10^{10}$ years). (Zurek, Joos & Zeh).

In more detail:-- Such models assume a factorization into 'object 'and 'environment', with the object having a small number of macroscopic/slow/massive degrees of freedom, while the environment has very (infinitely?) many microscopic/fast/light degrees of freedom. It is shown that under physically reasonable interactions, for any(?) initial universal state, the expansion of the universal state at later times, in terms of eigenstates of the object's macro/slow/massive variables, and their relative states:

$$\Psi(t) = \Sigma_i \; c_i(t) \; |\varphi_i> \otimes |E_i(t)>$$

has the property that the relative states of the environment , $|E_i(t)>$, become very rapidly very nearly orthogonal:

$$|<E_i(t)|E_j(t)>| = \exp(-\gamma t)\, \delta_{ij}$$

This means that the reduced state at t for the object (determined from $\Psi(t)$ by partial tracing over the environment) is very rapidly very nearly a mixture with components $|\varphi_i><\varphi_i|$.

Hence the summary as 'the weirdness of superposition is carried away by the environment'.

(b) Principles: But this success is *modulo* issues of principle about (i) having exponential 'tails' and (ii) having got an improper, i.e. not ignorance-interpretable, mixture.

One can respond to these models: Agreed, the interference terms being tiny implies the probabilistic indistinguishability of the actual and desired states. So statistics gathered in experiments will not refute the attribution of the desired state. 'For all practical purposes': Bell's FAPP. But as regards the individual case, and what is definite in it -- *close but no cigar!* The apparatus' reduced state is at no finite time, i.e. never!, a mixture in pointer position. (And even if it was a mixture, we could not give it the ignorance interpretation: it would be an 'improper mixture'.) That is: the orthodox eigenvalue-eigenstate link, applied to the total system, trivially dictates that the apparatus pointer has no definite position! (Indeed, in general, it dictates that the apparatus has no definite value for many quantities.) (Interesting contrast with thermodynamics.)

So we have to supplement these models with suitable principles for values, principles going beyond the eigenvalue-eigenstate link. We can get suggestions from the interpretations listed under the second strategy, (Values). (Zurek endorses the relative-state interpretation; Dieks combines decoherence with his modal interpretation.)

...Philosophy at last! ...

Or do we have to? The numbers are so low, so fast, for so long! Maybe recovering the success of classical physics does not need definite values, even of a few physically significant quantities like position. Maybe it just needs the appearance of definiteness.

It is hard to know; partly because, as the word 'appearance' hints, answering this question might push us back towards our experience, and its apparent definiteness. (NB: This is short for: 'the apparent definiteness of value, for a few physically significant quantities like position, in the contents of our experiences'. I take it that on any view of the measurement problem, it is the definiteness within the content, not of the experience itself, that matters.)

Being thus pushed back is no embarrassment. In any metaphysics, we need to 'close the circle'. Any description of the world that someone advocates as being complete, or as an outline of such a complete description, must include an account of us (and how we come to that description). So any fundamental physical theory must mesh with our experience as observers (Galileo, Hobbes, Helmholtz and modern psychophysics).

In any case:- See (Con) and (Mind) for two wild versions of the connection with experience; and (Pilot) and (GGRWP) for two tame versions.

(Physical Values) *Accept the thrust of the Cat argument; and keep the orthodox (unitary) dynamics of a strictly isolated system. Secure a definite macrorealm by positing new values, beyond those of the orthodox eigenvalue-eigenstate link;.*

Any such extra i.e. heterodox values will need to satisfy (at least!) three constraints:
(a) they need to be consistent with Sec.1, A (1) (i) and (ii) (e.g.no-hidden-variable proofs);
(b) they need to mesh with the empirical success of the orthodox Born-rule probabilities;
(c) they need to evolve st the definiteness of the macrorealm is maintained.

(Physical Values) includes some tame (worthy!) views: the hidden variable theory of Bohm, and the modal interpretations of Healey, Dieks, van Fraassen, Bub.
Some very different views fall under (Physical Values): two (comprehensible!) examples are:

(Modal) The modal interpretations of Kochen-Healey-Dieks, van Fraassen, and Bub. I will specialize to the first, to be called K-H-D; though as we shall see, they differ in important ways.

(1) The Schmidt (polar decomposition) theorem says that for an arbitrary state $\Psi \in H_1 \otimes H_2$ there are bases in the factor spaces st $\Psi$ can be expressed as: $\sum_i (c_i \varphi_i \otimes \psi_i)$

and furthermore in the generic case where $c_i \neq c_j$ for $i \neq j$, this expansion (the choice of bases in the factor spaces) is essentially unique.

(2) In their original presentations, K-H-D propose (i)-(iii), as follows:

(i) the component systems of a composite system in state $\Psi$ do have values for the quantities that have as their orthobases those bases appearing in the Schmidt decomposition, i.e. the bases $\{\varphi_i\}$ and $\{\psi_i\}$; (with degeneracy, the definite quantities will be the corresponding nonmaximal quantities).

(ii) the values possessed are the corresponding eigenvalues; and the value corresponding to the label i occurs with probability $|c_i|^2$. And there is a joint probability distribution for values of these Schmidt-selected quantities on both the component systems; namely, with trivial (i.e. 0 and 1) conditional probabilities. That is: given that the value corresponding to the label i occurs on the first component, the probability is 1 that corresponding value (corresponding to the label i) occurs on the second component.

(iii) the quantum state $\Psi$ evolves always according to the Schroedinger equation.

(3) The effect of (i) and (ii) can be obtained without appealing to the Schmidt decomposition, by instead using directly the reduced state of a system, determined by tracing out the 'other system'. Recall: This reduced state, in general mixed (and as discussed: an 'improper mixture') has a unique spectral resolution into orthogonal (in general multidimensional) projectors:

$$W = \Sigma_i \; w_i P_i$$

where the $P_i$ are projectors onto the vectors of the Schmidt decomposition (degeneracies apart). And the coefficients $w_i$ in this resolution are the $|c_i|^2$ of the Schmidt decomposition (or sums of these, allowing for degeneracies).

So the effect of (i) and (ii) can be obtained by proposing, in effect, the values and associated probabilities of values that are attributed by an ignorance interpretation of the 'improper mixture' W. That is, the effect of (i) and (ii) can be obtained by proposing:

(i') the component systems of a composite system in state $\Psi$ do have values for the (quantities represented as) projectors in the spectral resolution of the component's reduced state; and for functions of these projectors;

(ii') the values possessed are the corresponding eigenvalues; and the value corresponding to the projector $P_i$ occurs with probability $w_i$. Again, there is a trivial joint probability distribution, which can be written with a trace-formula of the usual sort.

Note two points about this proposal. (A) Dieks & Vermaas now work with this version.

(B) This proposal is not a howler, contradicting the fact that improper mixtures are not ignorance-interpretable. (Nor was the original proposal (i) -(iii).) For the proposal is *not* that an individual system described by W (if you prefer: in an ensemble described by W) must really be in a vector state in the range of one of the $P_i$; (and is so with probability $w_i$, so that W is an ignorance mixture. The proposal is that the system has quantum state W (*not* a component of it); but the system*also has* a value with an associated probability for a component of W (i.e. a value with an associated probability, *as if* W were an ignorance mixture). Thus is the eigenvalue-eigenstate link broken! Thus are the extra values supplied!

(4) Turning to assessment, recall first our three constraints on extra values, (a) to (c). First, (a). This was the need to be consistent with:

(i) the apparent impossibility of building ensembles that have precise values (or at least: sharply peaked distributions of values) for each of two quantities such as position and momentum:

(ii) the algebraic Kochen-Specker-style no-hidden-variable proofs.

(iii) the statistical Bell-style no-*local*-hidden-variable proofs.

On (i), suffice it to say here that there seems no problem, simply because the quantities for which extra values are here proposed mutually commute (for a single orthobasis, or Boolean subalgebra, is picked out in the state space of each component system); contrast (Pilot).

On (iii), suffice it to say here that Dieks and Healey argue that the EPR-Bell experiment exhibits a kind of holism but no superluminal action ...

On (ii): Beware that in some respects the authors' proposals differ. Thus Dieks' proposal is that (i)-(iii) hold [or (i'), (ii'), (iii) hold], for any factorization of the universal state space into two factor spaces. Healey postulates a physically distinguished factorization. This makes for differences about the meshing of the value-attributions arising from different factorizations of a composite Hilbert space, in particular about how these many value-attributions can avoid Kochen-Specker-style contradictions. I will just make two simple points about this.

(a) Some kind of holism (the details different on different versions) might arise in cases where the composite system has 3 (or more) components, giving three (or more) factorizations into a twofold tensor product of the sort that the original proposal (i)-(iii) uses.

(b) But there is another way that alternative twofold factorizations occur; e.g. a two-dimensional harmonic oscillator has Hilbert space $L^2(R^2) = L^2(R) \otimes L^2(R)$. But this Hilbert space is also, in an obvious notation, $L^2(x=y) \otimes L^2(x=-y)$. Similarly in finite-dimensional cases: recent work by Bacciagaluppi gives conditions in which a Kochen-Specker-style contradiction results.

(5) Constraint (b) was the need to mesh with the empirical success of the Born-rule probabilities. At first, this seems no problem: postulates (i) and (ii) are tailor-made to capture the empirical success of the naive Born-rule probabilities, when (i) and (ii) are applied to the composite system comprising the measured object and the measurement apparatus.

There is however a problem about these postulates (due to Albert & Loewer). It concerns inaccurate measurement: in short, for such measurements it seems that proposal (i) can make quantities very different from pointer-position have definite values. Similarly for accurate but disturbing measurements, i.e. measurements in which the object-system states $\varphi_i$ are sent to (in general non-orthogonal) $\varphi'_i$ under measurement. Dieks argued that this problem can be overcome, i.e. pointer-position can be rendered definite, by invoking the interaction of apparatus with environment, i.e. by invoking decoherence. Bacciagaluppi & Hemmo vindicate this in detail.

(6) Constraint (c) was that the extra values need to evolve over time so as to maintain the definiteness of the macrorealm. But nevermind the 'so as to maintain...'! It is clear that the proposals above say *nothing* about the evolution of the extra values over time in an individual system, nothing about 'trajectories'. There is not even some postulate about two-time conditional probabilities.

In other words: given a $w_i = |c_i|^2$ distribution of an ensemble among the extra (say Schmidt-selected) values selected at one time, one has to redistribute the ensemble among the extra values selected at another time (in general, values of a very different quantity, with a very different distribution). And the problem is: between any two times, there are any number of ways to do the redistribution. One can imagine randomly redistributing between two times.

This topic is largely unexplored, and most agree that this is the biggest lacuna facing the modal interpretation. This topic also resonates! In four ways! It connects with two other interpretations, which I will deal with more fully below. So here, just a word for each:

(i) We will see that the many-minds interpretation faces a exactly parallel lacuna, concerning the time-evolution of its extra mental values.

(ii) And the relative-state interpretation, at least on some construals, simply 'outfaces' the lacuna; denying that there are any facts about individual system's time-evolution. (About minds, this seems incredible.)

And the topic of trajectories also connects with the consistent histories and quantum state diffusion formalisms: which (I maintain!) are not interpretations of quantum theory in the sense of solving the measurement problem. So I sketch them, and spell out the connection to trajectories in the modal interpretation, in (7) and (8) respectively.

(7) Consistent Histories:--The Born-probabilities have a multi-time generalization: which we can therefore compare with any proposals for trajectories in the modal interpretation.

The generalization is given by a trace-formula with a time-indexed string of projectors, a formula called the decoherence functional (Groenewold, Feynman & Vernon, Gell-Mann, Griffiths...).

This is the core of the consistent histories approach. A sequence (or: string) of projectors, each projector indexed by a time, is called a 'history'. Because of 'interference terms', 'non-classical probability', applying the trace-formula to each time-indexed string of projectors in some arbitrary set of time-indexed strings, will lead to violation of the probability sum-rule. A set of time-indexed strings of projectors with the property that any of its members, taken together, obey the probability sum-rule is called a 'consistent (or: decoherent) set of histories'. (So NB: consistency is an adjective for a set.)

So the obvious comparison between the modal interpretation and consistent histories is as follows:

(i) take the projectors for a component system corresponding to the extra values posited by the modal interpretation (call them the 'Schmidt projectors');

(ii) form finite strings of them by selecting each of some finite set of times; i.e. apply the Schmidt theorem, or equivalently (i') of (3) above, at each time; (call each string a 'Schmidt path');

(iii) ask: is the set of all Schmidt paths a consistent set of histories?

The answer is: No. (Albrecht, Kent.)

This need not be trouble for modalists! The consistent-histories trace-formula is a very special multi-time distribution, though mathematically very natural in QT formalism (quadratic in the state-vector, like the original Born-rule.) So modalists should keep investigating ...

(8) Quantum State Diffusion:- Recall (B) of (3): that the modalist's extra values are not a howler, contradicting the fact that improper mixtures are not ignorance-interpretable. In detail, again: For the modalist's proposal is *not* that an individual system described by W must really be in a vector state in the range of one of the $P_i$. The proposal is that the system has quantum state W (*not* a component of it); but the system*also has* a value with an associated probability for a component of W (i.e. a value with an associated probability, *as if* W were an ignorance mixture). We are now facing the issue of trajectories for these extra values ...

Another research tradition, Quantum State Diffusion, with utterly different origins, is curiously similar. It is similar in also attributing values (in its case always represented by vector-states i.e. one-dimensional projectors, never by many-dimesnional projectors) that seem, naively, to be a howler contradicting the fact that improper mixtures are not ignorance-interpretable.

But the similarity is curious in that as regards attributing values and describing their trajectories, Quantum State Diffusion and the modal interpretation are in exactly complementary positions. Thus: Quantum State Diffusion has the trajectories (galore!), but not the values given by the Schmidt theorem, or equivalently (i') of (3) ... Let me explain!

(i) In classical mechanics we get a neat meshing of the equation describing the evolution of an individual open system and the equation governing an arbitrary statistical ensemble of such individuals. The classic case is Brownian motion. We have;

[Ind] an equation for the individual, with a term representing the influence of environment; the term is stochastic, because the environment is unknown/uncontrolled (a particular realization of the stochastic term represents a particular 'way the environment could go'). The Langevin equation: $d^2x/dt^2 + \Gamma(dx/dt) = F(t)$. Here F(t) is the stochastic term ('white noise').

[Stat] an equation for an arbitrary statistical ensemble of such individuals. The 'ensemble' is ideal in that all the various ways the environment could go, and their associated probabilities, are catered for; are built into the equation. This ideality washes out the indeterminism: the equation for the whole ensemble (consisting of open individual systems evolving indeterministically) is deterministic! Usually the individual system is given just by position, e.g. in Brownian motion so that ensemble is represented by $\rho(x)$. (Thus: diffusion equations, e.g. Fokker-Planck for Brownian motion: contrast the usual physical model of the heat equation, namely that $\rho(x)$ has an individual interpretation -- the amount of a fluid, caloric!)

(ii) In quantum mechanics [as usually presented!], we do not get a neat meshing of these two levels of description, between the evolution of an individual open system and the equation governing an arbitrary statistical ensemble of such individuals. All we get is the latter: master equations (diffusion equations), derived from postulating a Hamiltonian for the total system and then tracing out the environment. I.e., we get an equation for the reduced density matrix; pure states typically evolve to mixtures. So: a lacuna!

Quantum State Diffusion fills this lacuna, for the central case of Lindblad equations (roughly: Markovian master equations). QSD does this, using the familiar device of white noise stochastic terms to represent the influence of environment. (Gisin, Percival; Spiller.)

That is: QSD have an equation-schema, EqS say, for evolution of individuals i.e. evolution of vector-states (EqS having white noise terms) st:

> Given any Markovian master equation, MEq say, governing an arbitrary reduced density matrix $\rho(t)$, QSD will derive (?check) from MEq a set of operators $\{L_i\}$ ('L' for Lindblad) on the open system's Hilbert space such that: putting the $L_i$ into EqS, together with a postulated set of corresponding white noise terms $\xi_i$:
>
> > The evolution by MEq can be regarded as mere averaging over individual evolutions by EqS. (The averaging is done with the probabilities of the white noise stochastic process: probabilities that are 'a priori', i.e. independent of total hamiltonian, of Meq, of $\rho(t)$.) Let us write **M**( ) for this averaging: and let us

write $|\psi_\omega(t)\rangle$ for the individual 'quasi-state' driven by a particular realization $\omega$ of all the stochastic processes $\xi_i$, according to EqS. So the result is:-

All t: $\rho(t)$ [as driven by MEq] = $\mathbf{M}(|\psi(t)\rangle\langle\psi(t)|) = \Sigma_\omega \text{ prob}(\xi_i = \omega).|\psi_\omega(t)\rangle\langle\psi_\omega(t)|$

(iii) Impressive models, computer simulations etc arise from this! Further, one can apply the approach to the usual von Neumann model of measurement, and thus 'recover' the projection postulate. That is: there are models of measurement interaction which give the effect of the projection postulate, sending an initially pure $\rho(t)$ for the measured system, $\rho(t)$ with amplitudes $c_i$ for the measured observable, into a final mixture of eigenstates of the measured observable with weights $|c_i|^2$. (E.g. measurement of the energy of an oscillator. The model has the $L_i$ as you would expect: $L = a\dagger a$.)

(iv) Several conceptual issues arise, internal to QSD. Very briefly:

($\alpha$) The maths and the concepts are very similar to GRW-Pearle: compare and contrast! For example, look at what QSD says happens to a pointer. (This is the other 'half' of the scenario in (iii); indeed, it is the half we are more concerned to get into a final mixture - who cares what weird improper mixture some micro-system might get into?!)

($\beta$) Recall the non-uniqueness of the decomposition of mixtures! QSD's decomposition of $\rho(t)$ into $\Sigma_\omega \text{ prob}(\xi_i = \omega).\ |\psi_\omega(t)\rangle\langle\psi_\omega(t)|$ is 'physically preferred' in at least this sense:

that the $\xi_i$ and Li represent the influence of environment (with all the 'uncontrolled' elements of the physics of the environment cleverly packed into the $\xi_i$ !); and the $\text{prob}(\xi_i = \omega)$ come straight out of the stochastic processes textbooks!

But $\rho(t)$ can in general be decomposed in many ways, e.g. into its spectral resolution. This suggests:-

EqS may well not be the unique 'individual-evolution underpinning' for MEq. By considering different arbitrarily chosen decompositions of $\rho(t)$, as driven by MEq, for different times, and inventing prescriptions about how an individual case should transit over the time-interval $t_2 - t_1$, from an element of the $t_1$-decomposition to an element of the $t_2$-decomposition, we could presumably write down many pretty arbitrary 'individual-evolution underpinnings' -- even if we impose constraints like some kind of continuity ... (The building of $\rho(t)$ for the ensemble from these individual histories need not, a priori, use textbook white-noise probabilities).

And among all these individual-evolution underpinnings, there may be one that is physically reasonable in as good a sense as that of EqS. The obvious suggestion is that (whatever one says about the prescriptions for transiting), it is physically reasonable to take at all times the spectral resolution of $\rho(t)$. Thus we have been led back again to ...

(v) ... trajectories in the modal interpretation!

Consider first the generic simple case without degeneracy during the time interval considered: i.e. all the Schmidt projectors are, at all times,one-dimenisonal. So in effect an orthobasis evolves continuously (and determinsitically given the total Hamiltonian) in the component system's Hilbert space. (In case it helps you!: for each t, there is a unitary map U(t), but there is no single H such that $U(t) = \exp(iHt)$; rather for each t there is merely an H(t) such that $U(t) = \exp(iH(t))$ ... behold the significance of 'Ht' vs. 'H(t)' !)

Suppose a modalist postulates (cf. end of (iv); and -- cf. end of (7) -- who cares about the inconsistent histories!): ($) an individual system's values follow the Schmidt paths.

($) clearly violates the original modalist proposal to have at each time the extra values enjoying a $|c_i|^2$ distribution. Can the modalist just live with this, i.e. drop the original proposal, and hope to get the empirical success of the Born rules as merely a statement about measurement results?

Maybe! But the obvious place to look for details of how to render the Born-rules phenomenological like this is QSD. And since in (iii) above, we never used a spectral resolution, it isn't clear that QSD can help.

In other words, the problem is: QSD will drive *its* vector-states, occurring in *its* favourite decomposition of $\rho(t)$, namely $\Sigma_\omega$ prob$(\xi_i = \omega)$. $|\psi_\omega(t)\rangle\langle\psi_\omega(t)|$ , into a mixture that smells like it vindicates the Born-rule phenomenologically. (And as ($\alpha$) in (iv) says, maybe this can also be done for what is dear to our hearts -- the pointer.) But it is not at all clear (as they say in Oxford philosophy) that QSD techniques can similarly drive the modalist's vector-states, the modalist's favourite decomposition of $\rho(t)$, into a mixture that smells like it vindicates the Born-rule phenomenologically ...

(vi) Coping with degeneracies. Here ($) may well survive nicely:--

Recent work by Bacciagaluppi & Vermaas shows that the Schmidt paths of a two-dimensional system interacting with an N-dimensional enviroment can be continued through 2-dimensional degeneracy subspaces. (The limit is the same from both sides.)

... and 3-dimensional degeneracy subspaces are in some sense of measure zero ...

ble with the trajectory (5), and hence it makes no sense to ask whether one of them occurred "instead of" some history based upon (5). Quantum mechanics allows us to tell a variety of different stories about a system, but a question of the form "Which of these *really* took place?" asked in terms of comparing two mutually incompatible histories, makes no sense quantum mechanically.

If the second beam splitter is inserted, the trajectory corresponding to (2) is now

$$\epsilon C_A C_B \to (\alpha+\beta) C_A C_B/\sqrt{2} \to C_A^t C_B. \tag{7}$$

Once again, this is a perfectly acceptable trajectory, and it implies that the history which ascribes a superposition state to the particle at $t_2$, and that $C_A$ (and not $C_B$) has been triggered by the particle at time $t_3$, occurs with probability 1. On the other hand the two trajectories

$$\epsilon C_A C_B \to \begin{cases} \alpha C_A C_B \to (C_A^t C_B + C_A C_B^t)/\sqrt{2}, \\ \beta C_A C_B \to (C_A^t C_B - C_A C_B^t)/\sqrt{2}, \end{cases} \tag{8}$$

also form an elementary family, and each can be assigned probability 1/2. Now while there are a variety of reasons why the practical minded physicist will usually prefer to discuss (7) rather than (8), which corresponds to histories which have no very obvious laboratory interpretation, there is no reason in principle why (8) must be excluded from consideration, any more than (6). Quantum mechanics does not tell us that (7) must occur rather than one of the possibilities in (8); it simply tells us that asking whether one occurs rather than the other makes no sense.

The relation of quantum trajectories to the consistent histories approach to quantum mechanics arise through the fact that the noninterference condition following (3) is, in effect, a type of consistency condition. Unlike the consistency conditions of Omnès [7,8] and the decoherence conditions of Gell-Mann and Hartle [9,10], the noninterference condition is manifestly time symmetric: it is unchanged if the time axis is reversed. But it also differs from the time-symmetric condition of Griffiths [5] in that the initial and final times play no distinguished role. By means of the result stated below (4), one can show that the "generalized records" which Gell-Mann and Hartle [10] obtain by applying their "medium decoherence condition" to a *pure* initial state are the same as the elementary family of quantum trajectories associated with this state, given a suitable choice of the bases at later times. In the case of a density matrix at the initial time, the noninterference condition is a particular instance of what Gell-Mann and Hartle call the "medium-strong decoherence condition," when that is appropriately interpreted. The noninterference condition is definitely stronger than the consistency conditions employed by Griffiths and by Omnès. However, many of the specific examples considered by these authors (see, e.g., the discussion of the EPR paradox in [6]) satisfy the stronger condition, and thus can be discussed equally well using quantum trajectories, with identical results. It is also possible that there is a satisfactory alternative definition of quantum trajectories employing a noninterference condition which is weaker than that used in this paper.

The line of thought presented here began during a workshop held at the Aspen Center for Physics during June of 1992. I am indebted to several of the participants, in particular M. Gell-Mann, J. Hartle, R. Omnès, W. G. Unruh, and W. Zurek, for some very lively and useful discussions. Financial support for this research has come from the National Science Foundation through Grant No. DMR-9009474.

[1] A convenient summary of the history of quantum mechanics will be found in M. Jammer, *The Philosophy of Quantum Mechanics* (Wiley, New York, 1974).

[2] A very useful collection of reprints or translations of original papers will be found in *Quantum Theory and Measurement*, edited by J. A. Wheeler and W. H. Zurek (Princeton Univ. Press, Princeton, 1983).

[3] There is an enormous literature on this subject. Quite readable discussions will be found in: N. D. Mermin, Am. J. Phys. **49**, 940 (1981); **58**, 731 (1990); F. Selleri, *Quantum Paradoxes and Physical Reality* (Kluwer Academic, Dordrecht, 1990), among others.

[4] See E. P. Wigner, Am. J. Phys. **31**, 6 (1963), reprinted in [2], for a very clear presentation.

[5] R. B. Griffiths, J. Stat. Phys. **36**, 219 (1984); in *Fundamental Questions in Quantum Mechanics*, edited by L. M. Roth and A. Inomata (Gordon and Breach, New York, 1986), p. 211; in *New Techniques and Ideas in Quantum Measurement Theory*, edited by D. M. Greenberger (New York Academy of Sciences, New York, 1986), p. 512.

[6] R. B. Griffiths, Am. J. Phys. **55**, 11 (1987).

[7] R. Omnès, J. Stat. Phys. **53**, 893 (1988), and items cited in [8].

[8] R. Omnès, Rev. Mod. Phys. **64**, 339 (1992).

[9] M. Gell-Mann and J. B. Hartle, in *Complexity, Entropy, and the Physics of Information*, edited by W. Zurek (Addison-Wesley, Reading, 1990), and items cited in [8].

[10] M. Gell-Mann and J. B. Hartle, in *Proceedings of the Twenty-Fifth International Conference on High Energy Physics, Singapore, 1990*, edited by K. K. Phua and Y. Yamaguchi (World Scientific, Singapore, 1990); (to be published).

[11] See [8], Sec. II.C.